Parametric Amplifier Noise Analysis

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Abstract—**The effect of a different value of series resistance associated with the varactor in its imbedding at each of the important frequencies present in a one-port parametric amplifier is analyzed. The modified equations for effective noise temperature, amplifier gain and pump power required thus obtained are compared to experimental values measured on a 4.8 Gc/s parametric amplifier.**

Finally a simple and precise method for adjusting the amplifier to get close correlation between the measured varactor parameters using Kurokawa's [1] method at signal idler, and pump frequencies and the overall amplifier performance is presented.

I. Introduction

IN GOING from the extensive literature on the theoretical analysis of the varactor diode parametric amplifier to the design and construction of a working model, the engineer is required to bridge the sometimes wide gap between the idealized model and the "real world."

This paper is written to develop a refinement in the theory which can take account of an important discrepancy often present in the practical application. Experimental techniques for the application of this more exact theory are then presented with some results.

There are three approaches open for the construction of a circuit model for the parametric amplifier.

Firstly, a continuous model could be attempted which could account for the circuit behavior of the varactor over a wide frequency range, similar to the Hybrid π model of the transistor which is useful up to half the alpha cutoff frequency. Certainly such a circuit model could be constructed, but would prove too cumbersome in use.

Secondly, a very simple circuit model, such as the generally used series resistance-capacitance to represent the varactor junction, can be employed.

The figure of merit used for the varactor is the cutoff frequency

$$\omega_c = \frac{1}{C_{vb}R_k} \tag{1}$$

where C_{vb} is the capacity at reverse breakdown and R_k is the series resistance of the varactor. This is assumed to be a constant over the entire useful frequency of the varactor.

The third approach, which is the subject of this paper, is to use the simple series model, but to obtain exact information about the circuit values to be used by measurement at the discreet frequencies of interest.

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There is nothing absolute about any measurement of cutoff frequency, because the varactor junction can never be completely separated from its imbedding and the effect of imbedding coupling and losses is largely frequency dependent. Thus, the use of a different value of cutoff frequency as measured at the different discreet frequencies of interest is fully justified. The technique introduced by Kurokawa [1] for the measurement of cutoff frequency of a varactor in its parametric amplifier circuit, becomes a powerful tool for establishing this variation of cutoff frequency with measuring frequencies. Using the information obtained from this measurement at signal frequency ω_s, pump frequency ω_p, and idler frequency ω_i, it is possible to get very accurate prediction of parametric amplifier performance, provided that a suitable modification to the theory is made. Penfield and Rafuse [2] have provided a clear and orderly derivation of the theory of parametric amplifier operation, and for convenience, their analysis will be followed closely in Section IV.

Possibly the main reason that the Kurokawa method of varactor characterization was not immediately taken up by varactor manufacturers, is due to the fact that the measurement includes the circuitry in which the varactor is embedded in an intimate way. This is desirable however, from the point of view of the design engineer. By measurement of each of the important frequencies in the parametric amplifier, signal frequency ω_s, pump frequency ω_p, and idler frequency ω_i, the varactor characteristics in a particular circuit are completely known, and the performance of the parametric amplifier can be accurately predicted; such as its noise figure, pump power, and the required signal source impedance.

When measured in its circuit, the varactor can be thought of as being seen through an imperfect, lossy transformer, all parts of the varactor being equally affected. Without much loss of generality, C_j the junction capacity of the varactor can be assumed as being constant at its dc value, and the variation of ω_c can be explained by variation in R_k.

When the absolute transformed value of R_k is needed, such as in gain calculations, it can be measured directly in terms of signal frequency input VSWR.

II. Circuits, Models, and Assumptions

The parametric amplifier to be analyzed is the negative resistance, oneport type. The varactor is represented by a series RC circuit as shown in Fig. 1. A practical parametric amplifier circuit which is a suitable model for the analysis is shown in Fig. 2.

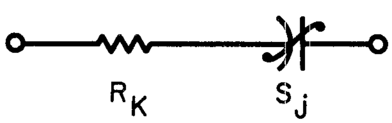


Fig. 1. Varactor circuit model.

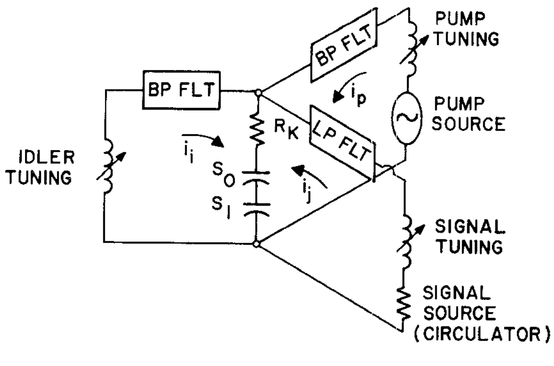


Fig. 2. Parametric amplifier circuit showing time varying elastance.

The basic assumptions are that the pumping is by current flow at ω_p, that the signal introduced as a current flow, and that an idler current flows but that only a negligible upper sideband ω_u current flows. Any series resistance component associated with the external series circuits cannot be distinguished from the series resistance of the varactor. Different values of series resistance can be presented at different frequencies, whether due to deliberate idler circuit loading, or due to varactor case and mounting losses.

Let R_s be the total value of series resistance associated with S_j at ω_s, similarly R_i at ω_i, and R_p at ω_p.

III. Varactor Parameters

As shown in Fig. 1 the varactor is assumed to consist of a series resistance and a nonlinear elastance.

The voltage dependence of the varactor is written as

$$S = S_{\max}\left(\frac{V + \Phi}{V_B + \Phi}\right)^{\gamma} \tag{2}$$

where V_B is the breakdown voltage, Φ is the contact potential and γ is the voltage law for the junction.

Under the influence of a periodic pump, the elastance time function may be expanded in a Fourier series.

$$S_{(t)} = S_0 + \sum_{-\infty}^{\infty} S_n e^{jn\omega_p t} \tag{3}$$

or

$$S_{(t)} = S_0 + S_1 + S_1^* + S_2 + S_2^* \cdots \tag{4}$$

S_n and S_n^* are half amplitude Fourier coefficients for positive and negative frequencies respectively.

The ratio

$$\frac{|S_n|}{S_{\max} - S_{\min}}$$

describes the degree of pumping for any harmonic of the pump, and is referred to as the modulation factor m_n.

The cutoff frequency of the varactor is defined as

$$\omega_{ck} = \frac{S_{\max} - S_{\min}}{R_k} \tag{5}$$

and

$$m_1\omega_{ck} = \frac{S_1}{R_k} \tag{6}$$

where the subscript k denotes the frequency at which the cutoff frequency is measured.

IV. Analysis and Results

The starting point of the analysis is to write voltage across the varactor for the circuit in Fig. 2 as

$$V_{(t)} = R_k i_{(t)} + S_{(t)} \int i_{(t)} dt + e_n \tag{7}$$

The small signal assumption made here is that the elastance time function is strictly controlled by the pumping and that the small signal currents do not affect it. Hence, $S_{(t)}$ is taken outside the integral sign. e_n is the noise voltage due to R_k.

$V_{(t)}$, $I_{(t)}$, and $S_{(t)}$ are all expanded in Fourier series in a general way, without specifying the form of the nonlinearity. If the frequency terms are considered to be of the form $(n\omega_p + \omega_s)$ then V_i^* and V_s are sufficient. Equation (7) reduces to

$$V_i^* = \left(R_i - \frac{S_0}{j\omega_i}\right) I_i^* + \frac{S_1}{j\omega_s} I_s + e_{ni}^* \tag{8}$$

$$V_s = \left(R_s + \frac{S_0}{j\omega_s}\right) I_s - \frac{S_1}{j\omega_i} I_i^* + e_{ns} \tag{9}$$

The second terms in (8) and (9) are the important interaction terms. Signal current flowing through the time varying elastance gives rise to idler voltage, and idler current flowing through the time varying elastance produces signal voltage. The first term in each equation is "passive." Study of these two equations leads to the understanding of the parametric amplifier as a flow process as sketched in Fig. 3.

Equations (8) and (9) are solved, for the important terms, using external idler constraint, and the results of this solution are discussed here in detail.

The input resistance of a parametric amplifier with series tuned signal and idler circuits is given as follows:

$$R_{\text{in}} = R_s 1 - \frac{m_1^2 \omega_{cs}\omega_{ci} R_i}{\omega_s \omega_i (R_i + R_{xi})} \tag{10}$$

where R_{xi} is the external series idler resistance.

The equivalent noise temperature for a parametric amplifier which is at physical temperature T_d, using external idler resistance which is at temperature T_{xi} is

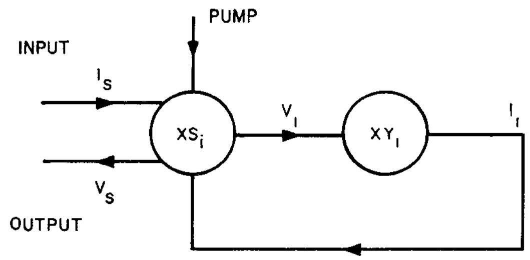


Fig. 3. Parametric amplifier flow process representation.

given:

$$T_n = \left(\frac{G-1}{G}\right) \cdot \left[\frac{R_sT_d + \dfrac{m_1^2\omega_{ci}^2R_i^2(R_iT_d + R_{xi}T_{xi})}{\omega_i^2(R_{xi}+R_i)^2}}{R_s + \dfrac{m_1^2\omega_{cs}\omega_{ci}R_sR_i}{\omega_s\omega_i(R_i+R_{xi})}}\right] \tag{11}$$

where G is the power gain.

For zero external idler resistance and high gain (12) reduces to

$$T_n = T_d\left(\frac{\omega_s}{\omega_i}\right)\left(\frac{m_1^2\omega_{cs}\omega_{ci} + \omega_i^2}{m_1^2\omega_{cs}\omega_{cs} - \omega_s\omega_i}\right) \tag{12}$$

Large, unexplained discrepancies can be present between the calculated and measured values of noise temperature if ω_{cs} and ω_{ci} are assumed to be the same. In practice ω_{cs} can be considerably less than ω_{ci} if the signal circuit contains losses due to idler and pump frequency separation filters. A discussion of this effect is included in Section V. The form of the variation of noise temperature with cutoff frequency is not easily seen from this equation, but usually numerical computations of noise temperature are most useful.

Another aspect which requires the recognition of the nonhomogeneous series resistance is in the calculation of the pump power required to achieve a given value of m_1. For sinusoidal current pumping, the pump power[1] in terms of m_1, and varactor parameters under self-bias operation for a graded junction varactor ($\gamma = 0.33$) is

$$P = \frac{(V_B + \Phi)^2}{R_p}\,\frac{\omega_p^2}{\omega_{cp}^2}\,\frac{3}{2}\,am_1 \tag{13}$$

where a is defined by the equation[2]

$$a = 2\left(\frac{V_0 + \Phi}{V_B + \Phi}\right)$$

where V_0 is the operating self-bias voltage. The value for ω cutoff to be used in this equation is that measured at ω_p, i.e., ω_{cp} does not enter the other equations for gain and noise, and if the circuit coupling has been optimized at ω_s and ω_i it is possible to have a relatively low ω_{cp} resulting in a much higher pump power requirement than expected.

In calculating power dissipation in the varactor, it must be recognized that not all of R_p is contained in the varactor. Varactor dissipation should be based on a measurement of ω_{cp} made in an optimized circuit.

For a given gain in the amplifier the value of m_1 is determined by the required value of R_{in}.

V. Design Method and Experimental Results

The way in which this theory can be used to predict the operation of a parametric amplifier accurately can best be illustrated by following through an actual design, with experimental results given.

The requirements were for a parametric amplifier at 4.8 Gc/s signal frequency with an overall noise temperature of 226°K when operated at 18-dB gain and room temperature. A circulator was available which had 0.4-dB insertion loss and 46-dB isolation.[3] This would account for 28°K noise temperature. The second stage noise temperature contribution based on a measured 10.1-dB noise figure (single sideband) and 18-dB gain is 42.6°K. This leaves a noise temperature for the varactor of 155.4°K.

A gallium arsenide diffused varactor[4] was available with a rated cutoff frequency of 234 Gc/s at −6 volts reverse bias. The junction voltage law γ and the contact potential Φ were found for this varactor by plotting the logarithms of capacity vs. bias voltage, Fig. 4. The pump frequency was chosen at 18.0 Gc/s after rough calculation based on the simple theory.

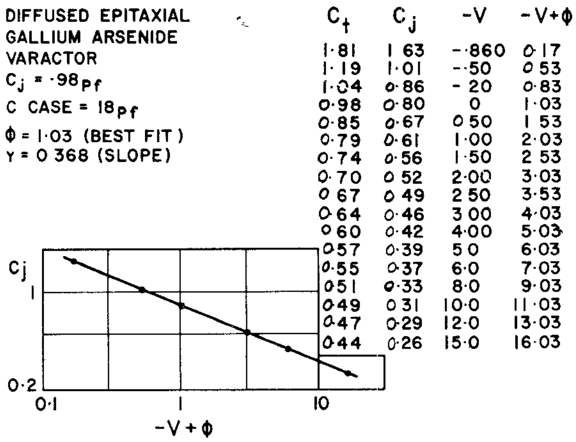
The cutoff frequency of the varactor was then measured in a suitable parametric amplifier circuit at signal and idler frequency using the method of Kurokawa [1].

The microwave configuration of this parametric amplifier is not the subject of this paper but a cross-sectional view is shown in Fig. 5 for completeness. Basically the requirements of a suitable circuit are that it has separation between its impedances at ω_s, ω_p and ω_i and also that it can tune out the average elastance of the varactor.

In this particular experimental amplifier, the idler circuit is available for direct measurement by removing the waveguide sliding short. If this were not the case, measurement could be made at the signal input after removal of the idler stop filter.

Briefly the Kurokawa measurement method consists of superimposing a low-frequency bias sweep on an operating point bias to cover the voltage range of the varactor which is of interest, and to analyze the signal, idler or pump frequency reflection coefficient for phase and amplitude information using a slotted line and oscilloscope display. The working cutoff frequency of the varactor can thus be determined.

[1] Penfield and Rafuse, [2], eq. (7.42), page 253.
[2] *Ibid.*, eq. (7.46), page 258.
[3] E&M Labs., type C202LPG serial (0).
[4] Sylvania, type D4057A (2).



C_t	C_j	-V	-V+Φ
1·81	1 63	-·860	0·17
1·19	1·01	-·50	0·53
1·04	0·86	-·20	0·83
0·98	0·80	0	1·03
0·85	0·67	0 50	1 53
0·79	0·61	1·00	2·03
0·74	0·56	1·50	2 53
0·70	0 52	2·00	3·03
0 67	0 49	2 50	3·53
0·64	0·46	3 00	4·03
0 60	0·42	4·00	5·03
0 57	0·39	5 0	6·03
0·55	0·37	6·0	7·03
0 51	0·33	8·0	9·03
0 49	0 31	10·0	11·03
0·47	0·29	12·0	13·03
0·44	0·26	15·0	16·03

Fig. 4. Varactor junction capacity vs. bias voltage.

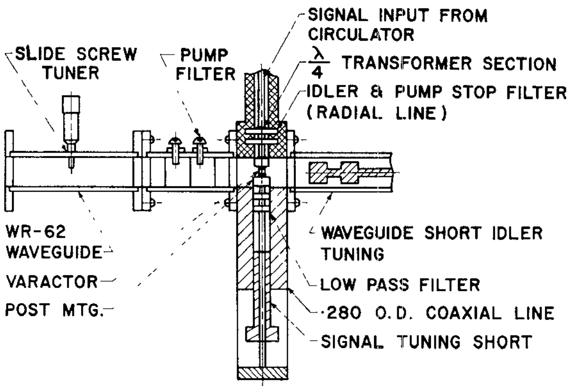


Fig. 5. Section view of parametric amplifier.

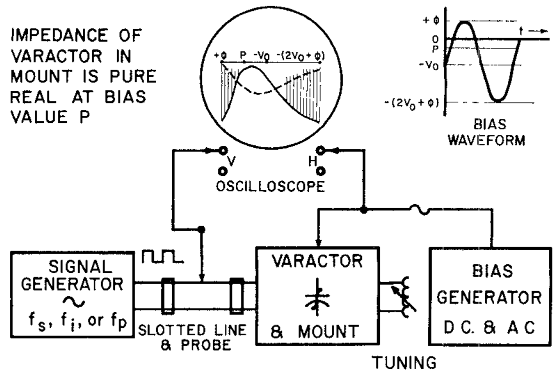


Fig. 6. Varactor measurement setup.

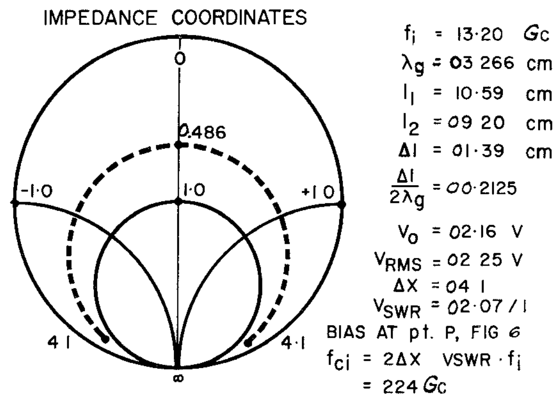


Fig. 7. Smith chart showing idler cutoff frequency ω_{ci} at 13.2 Gc/s.

The block diagram of the measurement setup is shown in Fig. 6.

The oscilloscope display shown is typical. On the horizontal axis, $-V_0$ is the operating point bias, P is the bias at which the circuit is tuned, $+\Phi$ is the forward excursion, $-(2V_0+\Phi)$ is the reverse excursion. The impedance of the varactor and circuit is real at bias point P. The phase shift of the reflection coefficient at points $+\Phi$ and $-(2V_0+\Phi)$ are measured and plotted on a Smith Chart as in Fig. 7.

Experience with this measurement shows it to be a powerful diagnostic tool. The circuits can be fully optimized using this method of presentation at each of the frequencies rather than taking them all together in the amplifier as they affect performance.

Using the tuning and circuits derived by this procedure, a prototype amplifier can be assembled and tried. The major adjustment of the circuit is that of the pumping level required for 18 dB of gain. Because of the nature of the measurement, a fixed (maximum) value of m_1 has been assumed and ω_{cs} and ω_{ci} have been measured at the bias sweep range corresponding to pump amplitude. The noise temperature is seen to have a strong dependence on the product $m_1^2\omega_{cs}\omega_{ci}$.

The signal source impedance is chosen [using (10) and (14)] for a desired degree of pumping to yield a value of $m_1^2\omega_{cs}\omega_{ci}$ which gives the noise temperature required [from (12)]. Whether the degree of pumping in a working amplifier is correct or not is ascertained by measuring the bias voltage developed across a self-bias resistor at the required 18 dB of gain.

The self-bias voltage which is generated across a bias resistor of about 2 megohms should be very close to the operating point bias used in the cutoff frequency measurement. The pumping amplitude for a given amplifier gain is adjusted directly by the signal source impedance.

A second setup of the tuning circuits is then made using this new bias as the operating point. The second prototype gives closely correlated results between the measured performance and the predicted performance.

Support for the assumption that pumping is by sinusoidal current is found in the close agreement in pump power required and input VSWR. The value of $m_1=0.22$ is found by assuming a linear interpolation between the values for m given by Penfield and Rafuse [2]. When $\gamma=0.33$, $m_1=0.212$ when $\gamma=0.5$, $m_1=0.25$; therefore when $\gamma=0.368$, m_1 will be very close to 0.22.

Calculation for input VSWR proceeds as follows: the voltage reflection coefficient is given by

$$\rho = \frac{R_g - R_{in}}{R_g + R_{in}} = 7.95 \tag{14}$$

where R_g is the source resistance at signal. Since

$$20 \log \rho = 18\text{-dB gain}$$

$$\frac{R_{in}}{R_g} = -0.776 \tag{15}$$

but R_{in}/R_s from (13) $= -9.32$. The input VSWR is R_g/R_s for the pump off conditions and bias adjusted to the tuned value.

$$\frac{R_g}{R_s} = \frac{\dfrac{R_{in}}{R_s}}{\dfrac{R_{in}}{R_g}} = \frac{9.32}{0.776} = 12.0{:}1 \tag{16}$$

It should be stressed that this is the value of input VSWR which should actually be measured in the amplifier. The only theoretical possibility for any deviation in the measured value is if m has been incorrectly evaluated. Input VSWR at signal frequency is considered the most absolute of the measurements. Since the input VSWR varies approximately as m_1^2, the discrepancy between the measured value of 12.8 and the calculated value of 12.0 would suggest an error in the 0.22 value of m_1 of -3.3 per cent.

If the calculations are based on the dc value of R_s found by assuming that the cutoff frequency measured at signal and idler frequencies is the same, or if the possibility of some transformation [3] of R_s due to case capacity when the signal frequency is close to the varactor series resonance is neglected, large errors could be introduced.

The measurement of pump power and also the measurement of pump cutoff frequency was made in the actual tuned pump circuit as shown in the block diagram of Fig. 8. By making both cutoff and power measurements under exactly the same conditions, good accuracy can be expected.

Noise temperature measurements are made by the $\mathfrak{Y}$ factor method of measuring noise figure using an AIL type 7010 noise lamp with Bendix TD39A argon discharge tube calibrated at 15.45-dB noise excess.

The estimated possible error in this measurement is made up as follows:

[0.1 dB = 7°K]	
Noise excess from lamp	±7°K
Coupling loss in mount	+0 −7°K
Attenuator Setting (reproducibility)	±3.5°K
Attenuator Calibration (relative)	±3.5°K
Circulator insertion loss	±3.5°K
Second stage contribution	negligible
Total possible error	+17.5°K −24.5°K

The results are given in tabular form in Table I.

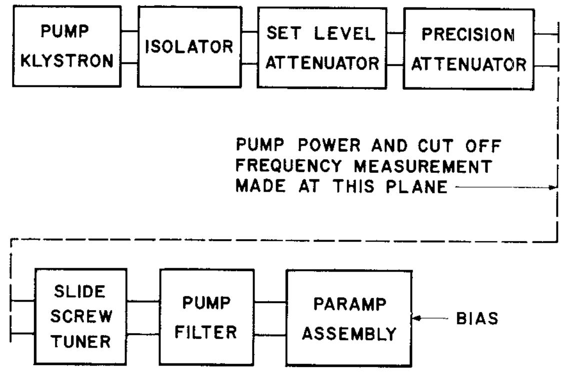


Fig. 8. Parametric amplifier and pump setup.

TABLE I
COMPARISON OF MEASURED AND CALCULATED RESULTS

Parameter	Measured	Calculated
gain	18.0 dB	
f_s	4.764 Gc/s	4.80 Gc/s
f_p	18.0 Gc/s	18.0 Gc/s
f_i	13.236 Gc/s	13.2 Gc/s
Bias dc	2.16 V	
ac rms Sweep	2.25 V	2.28 V
C_{j0}	0.80 pF	
ϕ	1.03	
γ	0.368	
m_1		0.22
f_{cs}	60.0 Gc/s	
f_{cp}	74.0 Gc/s	
f_{ci}	224.0 Gc/s	
Source resistance Z_0	50 ohms	
Pump power	129 mW +21.1 dBm	128 mW
Input VSWR	12 8/1	12.0/1
Effective noise temp. of circulator		28°K
Second stage noise temp.	2970°K	
Effective second stage temp.		42.6°K
T°		144.5°K
T_e for amplifier	203°K	215°K

VI. CONCLUSIONS

The most important conclusion to be drawn from this work is that the noise performance of a parametric amplifier can be predicted from the theoretical analysis of a simple model to within experimental measurement accuracy. This gives good support to the several assumptions made.

1) The simple series circuit of the varactor can be used with good accuracy if the value of R_k is allowed to take on different values at different frequencies.

2) Pumping can be accurately represented as sinusoidal current when the varactor mount is a simple resonance waveguide cavity of the type used in the model amplifier.

3) If the cutoff frequency is measured by Kurokawa's [1] method over the bias sweep range of $2(V_0+\Phi)$, where V_0 is the working self-bias voltage of the amplifier operated at the required gain, the value obtained is the exact value achieved in the amplifier "as pumped." Other methods for cutoff frequency require extrapolation and are less accurate.

4) Using the assumption of 3), the value of m_1 is always the "fully pumped value."

From the circuit equation an intuitive "flow chart" for the operation of a parametric amplifier has been developed, Fig. 3.

Finally a design method has been presented which exposes the contributing factors of parametric amplifier performance individually, allows their independent measurement, and gives information for the accurate prediction of the amplifier performance.

References

[1] Kurokawa, K., Use of passive circuit measurements for the adjustment of variable capacitance amplifiers, Bell Monograph 4089, Bell Telephone Labs., Inc., Nutley, N. J., 1961.

[2] Penfield, R , and P. Rafuse, *Varactor Applications*. Cambridge, Mass.: MIT Press, 1962. See especially Table 7.1, p 239.

[3] Hudson, A. C., Varactor self-resonance in parametric amplifier *Proc. IEEE* (*Correspondence*), vol 51, Sep 1963, pp 1248–1249

Phase Equalization Problems in a Phased-Array Transmitter

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Abstract—**Current radar systems employing programmed multiple antenna feeds to orient the antenna beam electronically require large numbers of parallel channels operating simultaneously with nearly identical phase-delay characteristics. These channels include active devices such as microwave amplifiers for which phase-delay, as well as gain and power output, must be identical to accomplish high pointing accuracy and maximum power addition. When the system is required to operate over a wide frequency band, all channels must track in phase.**

This paper describes problems encountered in the development of a prototype transmitter requiring multiple channel phase-delay equality. A new medium power pulsed TWT is described and measurements of its electrical length are discussed. Computer programming which reduced the vast amount of phase information derived from this development is described.

Phase Equalization

A BLOCK DIAGRAM of the transmitter under consideration is shown in Fig. 1. The maximum overall system phase error (about 35° rms) was established by the acceptable signal degradation (about 2 dB). A division of tolerances for the transmitter resulted in a 15° rms allowance for the power amplifier module.

Because the power amplifier modules must be interchangeable, it was required to equalize their electrical path lengths by referring their phase vs. frequency curves to an absolute reference and then compensating to obtain the best mean square fit to an average. Single frequency compensation may then be introduced to minimize wide-band phase tracking errors.

By maintaining close manufacturing tolerances on the several types of transmission line and on other passive devices (transitions, transformers, couplers, circulators, connectors, etc.) to minimize their VSWR, wide-band phase excursions are held to a minimum. Phase considerations for active devices, such as microwave power amplifier tubes, are more complex. Cumulative manufacturing tolerances for the many tube parts usually are not compatible with system phase requirements.

For instance, suppose we assume a standard or average phase characteristic as shown in Fig. 2. Assume another phase characteristic to be the curve for any tube, say the Kth tube. Let $\phi_{K,i}$ be the phase measured for this tube at frequency i. We wish to refer this curve to the standard such that the average value of its algebraic deviations, when overlayed on the standard within a specified frequency band, is zero. Expressed mathematically,

$$\phi'_{K,i} = \phi_{K,i} + \phi_{K,0} \tag{1}$$

where

$$\phi_{K,0} = \frac{1}{NJ}\sum_{i=1}^{J}\sum_{j=1}^{N}\phi_{j,i} - \frac{1}{J}\sum_{i=1}^{J}\phi_{K,i} \tag{2}$$

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